Representing Codes for Belief Propagation Decoding

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I. Introduction

The best codes of short or intermediate blocklength that have so far been discovered are usually defined in ways that do not immediately suggest a sparse parity check matrix representation. For this reason, large classes of classical textbook codes, which would give excellent performance under optimal decoding, have been mostly ignored as candidates for the belief propagation (BP) decoding algorithm. A notable exception are the one-step majority logic decodable codes, which have been shown to have excellent error-correcting performance when decoded using BP [1]. We report here on a method for generating sparse generalized parity check (GPC) matrix representations of other classical codes.

II. Improving GPC Matrices

An M by N *generalized parity check* (GPC) matrix representation [2] of an (n,k) binary linear block code will have $N \geq n$ columns, and $M \geq N-k$ rows, where the M rows span an $N-k$ dimensional sub-space.

The BP algorithm seems to work best on GPC matrices that have the following characteristics:

1. The number of ones in each row is small.
2. The number of ones in each column is large.
3. For all pairs of rows of the matrix, the number of columns that have a one in both rows is small; ideally zero or one.

We developed an algorithm that improves GPC matrices in all three of these characteristics; but the new GPC matrices also have additional auxiliary bits, and because there is no evidence from the channel to determine the value of these bits, they may cause performance to deteriorate on channels other than the binary erasure channel (BEC).

Given an M by N input GPC matrix H, our algorithm outputs an M' by N' GPC matrix H'. The basic idea behind the algorithm, which is described in detail elsewhere [3], is to re-write constraints involving large numbers of bits by using auxiliary bits that encode the parity of sets of bits. Using this "divide-and-conquer" approach, we try to minimize the number of bits involved in each constraint by re-writing the constraints in terms of sets of bits that are as large as possible. We also try to use as many redundant constraints as possible.

III. Results for a Euclidean Geometry Code

We present some empirical results obtained applying our method on a $(n=255, k=127)$ Euclidean geometry (EG) code. In the notation of [4], the code we studied has parameters, $m=4$, $s=2$, and $\mu=1$. In the geometric interpretation of this code, each bit corresponds to a point, there are $21 * 255 = 5355$ lines each consisting of four points, and there are also 5355 planes consisting of four parallel lines. This code can be represented by a redundant parity check matrix H with $M=5355$ and $N=255$, where each row has weight 16.

Applying our algorithm to this input GPC matrix H, we obtain an output GPC matrix H' with $M'=32130$ and $N'=5610$. The weights of the rows are either four or five. Results using BP decoding with these matrices on the BEC are shown in Figure 1, where we have also shown the performance of BP decoding of regular $(3,6)$ Gallager codes and maximum likelihood (ML) decoding of random linear codes with parameters $(n=256, k=128)$ [5].

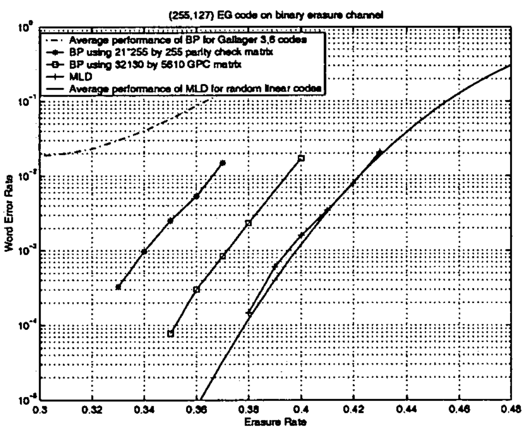

Figure 1: Performance of EG $(255,127)$ code on the BEC.

The results using BP decoding with H are already quite good–much better than BP decoding of Gallager codes. This is a consequence of using highly redundant parity check matrices with 5355 rows instead of just 128 rows. The performance using H' was even better, though still clearly distinguishable from ML decoding.

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Abstract

The best codes of short or intermediate blocklength that have so far been discovered are usually defined in ways that do not immediately suggest a sparse parity check matrix representation. For this reason, large classes of classical textbook codes, which would give excellent performance under optimal decoding, have been mostly ignored as candidates for the belief propagation (BP) decoding algorithm. A notable exception are the one-step majority logic decodable codes, which have been shown to have excellent error-correcting performance when decoded using BP. We report here on a method for generating sparse generalized parity check (GPC) matrix representations of other classical codes.

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Constructing Free Energy Approximations and Generalized Belief Propagation Algorithms

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Abstract

Important inference problems in statistical physics, computer vision, error-correcting coding theory, and artificial intelligence can all be reformulated as the computation of marginal probabilities on factor graphs. The belief propagation (BP) algorithm is an efficient way to solve these problems that is exact when the factor graph is a tree, but only approximate when the factor graph has cycles.

We show that BP fixed points correspond to the stationary points of the Bethe approximation to the free energy for a factor graph. We explain how to obtain region-based free energy approximations that improve the Bethe approximation, and corresponding generalized belief propagation (GBP) algorithms.

We emphasize the conditions a free energy approximation must satisfy in order to be a "valid" approximation. We describe the relationship between four different methods that can be used to generate valid approximations: the "Bethe method," the "junction graph method," the "cluster variation method," and the "region graph method."

The region graph method is the most general of these methods, and it subsumes all the other methods. Region graphs also provide the natural graphical setting for GBP algorithms. We explain how to obtain three different versions of GBP algorithms and show that their fixed points will always correspond to stationary points of the region graph approximation to the free energy. We also show that the region graph approximation is exact when the region graph has no cycles.

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